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EXPERIMENTAL DETERMINATION OF STRESSES IN DAMAGED COMPOSITES US--ETC(U)
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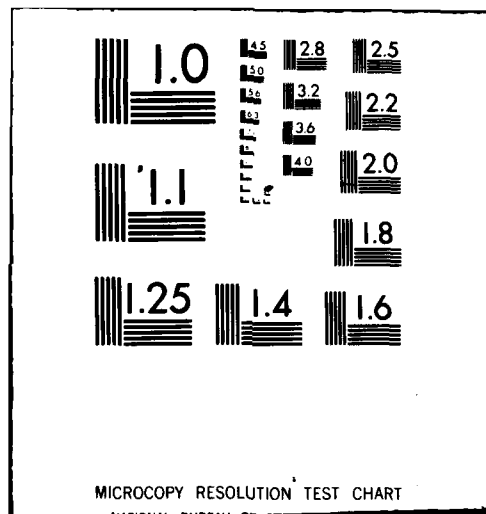
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EXPERIMENTAL DETERMINATION OF STRESSES IN DAMAGED COMPOSITES
USING AN ELECTRIC ANALOGUE

S. B. BATDORF

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Inadequate knowledge of the local stress distributions in damaged composites has been a major obstacle to progress in the understanding of damage accumulation and ultimate failure of such materials. Theoretical treatments of 3-dimensional uniaxially reinforced composites are difficult, and direct experimental observations of stresses around interior flaws are not feasible. An experimental determination of stress distributions can be made using an electric analogue. A scaled model of the composite including the damage		

20. Continuation of Abstract

is made with the fibers replaced by conducting rods and the matrix replaced by an electrolyte. The resistivity ratio of rods to electrolyte is taken equal to the elastic modulus ratio of matrix to fiber. A tensile force applied in the fiber direction is modelled by applying a potential gradient in the rod direction. The displacement distribution in the composite is then modelled by the potential distribution in the analogue to an accuracy somewhat better than that given by shear lag theory. Thus stress distributions can be found by measuring potentials in the analogue with the aid of an electric probe.

Experimental Determination of Stresses in Damaged
Composites Using an Electric Analogue

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Introduction

A few decades ago it began to be recognized that certain fibers such as glass, boron, carbon or graphite, and more recently kevlar could be prepared with several times the specific strength and stiffness obtainable in conventional structural materials. As a result a very substantial development effort was undertaken to capitalize on these properties. In spite of the extensive use now being made of fibrous composites, there is a widespread feeling that current applications still fall far short of the ultimate potential of such materials. The slow progress in practical application of composites is no doubt due in part to the fact that the theoretical foundations relating to the strength and stiffness of these materials have been very inadequate up to the present time. The stiffness of undamaged composites is pretty well understood even in the case of complicated laminated structures. But one of the great virtues of the long fiber composite is the fact that the tensile failure load greatly exceeds the load at which the first fiber fails. The stiffness of greatest concern in critical parts must therefore be stiffness of the damaged composite. Consequently as a practical matter, the determination of both the stiffness and the ultimate load on a composite require an understanding of the mechanics of damage accumulation. The most serious obstacle to progress in this area

is probably inadequate knowledge concerning the stress distribution in a damaged composite. This becomes evident from the following brief review of progress in this field.

The theory underlying progressive damage and ultimate failure of a collection of independent fibers, or a bundle, was worked out several decades ago by Daniels and Coleman [1, 2]. They showed that the strength of a bundle is substantially less than the average strength of the fibers of which it is composed. When a bundle of strong fibers is bound together by a relatively weak matrix to form a unidirectionally reinforced composite, the strength can be substantially improved. When a given fiber breaks, the load it no longer carries is transmitted to neighboring fibers as a result of shearing forces in the matrix, and some distance from the break it is transferred back into the original fiber, thus localizing the damage. Rosen [3] analyzed such a composite by conceptually dividing the composite into many slices, or mini-bundles, connected end-to-end. The length of each mini-bundle was taken to be the ineffective length of a broken fiber, or the shear transfer length. The load given up by a broken fiber was assumed to be equally distributed to all remaining fibers in a mini-bundle, and the theory of Daniels and Coleman was used to find its ultimate strength. Zweben [4] revised the theory to take into account the fact that the load given up by a broken fiber is mainly taken up by the immediate neighbors. In this model of damage accumulation the fiber adjacent to a break is more apt to fail than a distant fiber and this gives rise to the possibility of crack growth and eventual instability of the type first studied by Griffith in the case of homogeneous materials [5]. An effort by Zweben and Rosen [6]

to carry the analysis to the point of instability for composites was unsuccessful due to analytical complexities, and the authors proposed first occurrence of a double break (the theory for which they succeeded in working out) as a conservative estimate of composite strength.

Using an entirely different approach, Harlow and Phoenix [7, 8] obtained a virtually exact solution for the strength of a 2-D composite (a tape one fiber thick) on the basis of a highly idealized local load sharing rule. They assumed that the load given up by a series of broken fibers of any order was completely taken up by the two immediately adjacent neighbors, i.e., each of the two fibers adjacent to a break of any multiplicity takes 50% of the load. Hedgepeth [9] found that the proper figure is 33% for a single break and progressively less for higher order breaks. R. L. Smith [10] generalized the Harlow and Phoenix result and gave an approximate solution for the 2D problem for arbitrary stress concentration factors in the fibers adjacent to the crack. Batdorf [11] gave an approximate solution for a 3D fibrous composite for arbitrary stress concentration factors.

As a result of the difficulties associated with the analytical study of 3D composites, a number of authors have studied damage accumulation and ultimate failure using a Monte Carlo approach [10, 12, 13]. The studies that make use of specific local load sharing rules have usually made arbitrary assumptions in regard to stress concentration factors which generally tend to give too large a share of the total load to the immediately adjacent neighbors.

Calculations of the stiffness of damaged composites are similarly handicapped by lack of knowledge of stress distribution. Gottesman, Hashin,

and Brull have shown how to obtain estimates for upper and lower bounds on composite stiffness using a variational approach [14]. The upper bound employs the principle of minimum potential energy and makes use of an admissible displacement field. The lower bound is based on a consideration of minimum complementary energy, and employs an admissible stress field. The discrepancy between upper and lower bounds is approximately a factor 2 for $\alpha = 0.2$ where α is a measure of crack density per unit area.

The reason that available treatments of strength and stiffness of damaged composites are generally either rather crude approximations or are left in parametric form employing unknown parameters is the serious lack of knowledge concerning the stress distributions in damaged composites. The most widely employed source of information in the field is a paper by Hedgepeth and Van Dyke [15] in which local stress concentration factors are found for 3D uniaxially reinforced composites using shear lag theory. Unfortunately numerical results were only furnished for a very limited number of crack sizes (1, 2, 4, 9, 12, 16, etc. broken fibers) and for each crack size only one neighboring fiber was considered. Moreover, only the stress concentration factor was found, not the entire stress distribution. All the missing information could in principle be obtained using Hedgepeth and Van Dyke's equations but unfortunately they contain an undefined parameter Gh described only as the effective matrix shear stiffness. G is the matrix shear modulus but h is some unknown function of the fiber diameter, interfiber spacing and composite geometry (square, hexagonal, or some other type of array).

Recently Goree and Gross have applied Hedgepeth's equations to find detailed stress distributions for the 3D case [16]. In their analysis the matrix was conceptually divided into cells of square cross-section with a fiber at the center of each cell. One consequence of their model for material behavior is that the force transmitted by a fiber to one of its nearest neighbors is independent of the ratio of fiber diameter to fiber spacing. The authors recognized the approximate nature of this feature of their model, and in a later paper [17] they employed an effective shear stiffness derived from experimental data on boron aluminum. However, the general dependence of h on fiber volume fraction and geometry for 3D composites remains unknown at the present time.

An alternative approach for finding detailed stress distributions in damaged composites is to resort to experiment, but this also entails certain difficulties. It is obviously impractical to mount strain gauges on individual fibers. Optical techniques such as photoelasticity, Moire patterns and stress coat are largely limited to the study of surface phenomena.

Fortunately an experimental approach based on the use of an electric analogue can be employed. The analogue is related but somewhat superior to the shear lag approximation to composite behavior. Like shear lag, it assumes that all displacements are parallel to the fibers. Unlike shear lag, in the analogue the matrix carries its proper share of the direct stress.

Higgins' extensive review of electrical analogues to mechanics problems [18] lists one paper giving an electric analogue technique for solving shear lag problems [19], and at least one paper has been published since on the subject [20]. However, the approach in those papers is quite different from that proposed here. In References 19 and 20 the differential equations

of shear lag are replaced by difference equations, and a network of resistors is constructed to solve the resulting set of simultaneous linear algebraic equations. The continuous system is thus discretized, and the values of the various resistances have to be calculated in some way. The network becomes very complicated for a 3D composite.

The present approach uses a scaled model of the composite (including all damage) with conducting rods replacing the fibers and an electrolyte replacing the matrix. It will be shown that by proper scaling and choice of the resistivity ratio of rods to electrolyte the differential equations and boundary conditions for the potential in the electrical system are the same as those for the displacements in the mechanical system. Thus by measuring potential distributions in the electrical system with a probe, the strains and therefore the stresses can be found for the composite.

Equations Governing Composite Behavior

It will be assumed, as in shear lag theory, that all displacements are parallel to the reinforcing fibers. If the fibers are aligned parallel to the z-axis (see Fig. 1), then

$$u = v = 0 \quad (1)$$

The other main assumption of shear lag theory, that the matrix carries only shear, is not made here.

The displacements in the k'th fiber will be denoted by $w(x_k, y_k, z)$, where (x_k, y_k) are the coordinates of the center line of the fiber. Applying Hooke's law to the fiber

$$\sigma_z(x_k, y_k, z) = E_f \frac{dw(x_k, y_k, z)}{dz} \quad (2)$$

For the matrix material Hooke's law implies that

$$\tau_{zx} = G_m \gamma_{zx} = G_m \frac{\partial w}{\partial x} \quad (3a)$$

$$\tau_{zy} = G_m \gamma_{zy} = G_m \frac{\partial w}{\partial y} \quad (3b)$$

$$\sigma_z = E'_m \epsilon_y = E'_m \frac{\partial w}{\partial z} \quad (3c)$$

Here G_m is the shear modulus of the matrix material, and E'_m is the tensile stiffness of the material when lateral expansion and contraction are forbidden. It is related to Young's modulus E_m by the relation

$$E'_m = \frac{E(1-\nu)}{1-\nu-2\nu^2} \quad (4)$$

where ν is Poisson's ratio.

Equilibrium in the k 'th fiber requires that

$$A_f \frac{d\sigma_z(x_k, y_k, z)}{dz} = \oint (\tau_{zy} dx + \tau_{zx} dy) \quad (5)$$

where the integral is taken around the circumference of the fiber.

Using Hooke's law, the equation becomes

$$A_f E_f \frac{d^2 w_f(x_k, y_k, z)}{dz^2} = G_m \oint \left(\frac{\partial w_m}{\partial y} dx + \frac{\partial w_m}{\partial x} dy \right) \quad (6)$$

or

$$\frac{d^2 w_f}{dz^2} = \frac{G_m}{E_f A_f} \oint \left(\frac{\partial w_m}{\partial y} dx + \frac{\partial w_m}{\partial x} dy \right) \quad (7)$$

The boundary conditions are determined by the applied loading. If the rest length of the composite is L and it is extended to length $L + \Delta L$, then

$$w_f(x_k, y_k, \pm L/2) = \pm \Delta L/2 \quad (8)$$

If the fiber is unbroken the differential equation (7) and the boundary conditions (8) define its strain and therefore also its stress distribution. If it is broken at location z_{k0} the stress and strain at z_{k0} are zero. Thus each fiber segment obeys (7) and one of the boundary conditions (8). The other boundary condition becomes

$$\frac{dw_f}{dz}(x_k, y_k, z_{k0}) = 0 \quad (9)$$

Equilibrium of the matrix in the z -direction requires that

$$\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0 \quad (10)$$

Applying Hooke's law this becomes

$$G_m \left(\frac{\partial^2 w_m}{\partial x^2} + \frac{\partial^2 w_m}{\partial y^2} \right) + E_m \frac{\partial^2 w_m}{\partial z^2} = 0$$

or

$$\frac{\partial^2 w_m}{\partial x^2} + \frac{\partial^2 w_m}{\partial y^2} + \alpha^2 \frac{\partial^2 w_m}{\partial z^2} = 0 \quad (11)$$

where

$$\alpha^2 = E'_m / G_m \quad (12)$$

The end boundary conditions are the same as for the fibers

$$w_m(x, y, \pm L/2) = \pm \Delta L/2 \quad (13)$$

The side boundary conditions are a statement of continuity of w ,

$$w_m = w_f \text{ at all interfaces.} \quad (14)$$

Electric Analogue

For simplicity, let us initially consider an electric analogue in which fibers and matrix are replaced by a geometrically similar array of conducting rods immersed in a weakly conducting electrolyte. Fundamentally, the analogy exploits the similarity between Hooke's law and Ohm's law.

According to Ohm's law, in the k 'th rod the current density j is given by

$$A_r j_z(x_k, y_k, z) = \frac{d\phi(x_k, y_k, z)}{dz} (1/\rho_r) \quad (15)$$

where ϕ is the electrical potential and ρ_r and A_r are the resistivity and area of the rod respectively. In the electrolyte, Ohm's law states that

$$\vec{j} = \nabla \phi (1/\rho_e) \quad (16)$$

where ρ_e is the resistivity of the electrolyte.

Conservation of charge implies that in the rod

$$A_r \frac{dj_z}{dz} = \oint (j_{ey} dx + j_{ex} dy) \quad (17)$$

Here the integral is to be taken around the periphery of the rod, and j_{ex} and j_{ey} are the x and y components of the current density in the electrolyte. Applying Ohm's law, the equation can be written

$$(A_r/\rho_r) \frac{d^2\phi_r}{dz^2} = \frac{1}{\rho_e} \oint \left(\frac{\partial\phi_e}{\partial y} dx + \frac{\partial\phi_e}{\partial x} dy \right) \quad (18)$$

or

$$\frac{d^2\phi_r}{dz^2} = \frac{\rho_r}{A_r \rho_e} \oint \left(\frac{\partial\phi_e}{\partial y} dx + \frac{\partial\phi_e}{\partial x} dy \right) \quad (19)$$

The boundary conditions for the k'th rod are

$$\phi(x_k, y_k, \pm L/2) = \pm \phi_0/2 \quad (20)$$

where ϕ_0 is the potential difference between the rod ends. If the rod is broken at location z_{k0} equation (19) applies to each segment. In this case one boundary condition of type (20) applies while the other is obtained by noting that at the break the current is zero, as a result of which (using (15))

$$\frac{d\phi_r}{dz} (x_k, y_k, z_{k0}) = 0 \quad (21)$$

The differential equation for the electrolyte is obtained by noting the fact that for steady currents

$$\nabla \cdot \vec{j}_e = 0 \quad (22)$$

Applying Ohm's law (14) we obtain

$$\nabla^2 \phi_e = 0 \quad (23)$$

The end boundary conditions are

$$\phi_e (x, y, \pm L/2) = \pm \phi_0/2 \quad (24)$$

while the side boundary conditions are

$$\phi_e = \phi_r \quad (25)$$

at every interface.

In comparing the differential equations and boundary conditions for rods and electrolyte with those for fibers and matrix, we see that the only flaw in the analogy is in the matrix and electrolyte differential equations, (11) and (23) respectively. This discrepancy can be eliminated by using a scale factor for the z dimension differing from that common to the x and y dimensions. Let us introduce the coordinate

$$s \equiv \alpha z \quad (26)$$

Then (23) becomes

$$\frac{\partial^2 \phi_e}{\partial x^2} + \frac{\partial^2 \phi_e}{\partial y^2} + \alpha^2 \frac{\partial^2 \phi_e}{\partial s^2} = 0 \quad (27)$$

With the transformation (26), (19) becomes

$$\frac{d^2 \phi}{ds^2} = \frac{\rho_r}{\alpha^2 A_r \rho_e} \oint \left(\frac{\partial \phi_m}{\partial y} dx + \frac{\partial \phi_m}{\partial x} dy \right) \quad (28)$$

The end boundary conditions become

$$\phi(x, y, \pm \alpha L/2) = \phi_0/2 \quad (29)$$

while the side boundary conditions retain their previous form:

$$\phi_e = \phi_r \quad (30)$$

at all rod-electrolyte interfaces. Thus the electrical system is a faithful analogue of the mechanical system provided that the resistivity ratios are chosen appropriately.

To see how the choice is to be made we note that if the ratio of rod diameter to fiber diameter is K , the point (x,y,z) in the composite corresponds to point $(Kx,Ky,\alpha Kz)$ in the electric analogue. What is needed is a choice that will make

$$\phi \frac{(Kx,Ky,\alpha Kz)}{\phi_0} = \frac{w(x,y,z)}{\Delta L} \quad (31)$$

for all values of x, y , and z . A little reflection will convince the reader that with this choice the integrals in (7) and (28) are equal.

If we were to equate the factors in front of these integrals we would find

$$\frac{1}{\phi_0} \frac{d^2 \phi}{ds^2} = \frac{1}{\Delta L} \frac{d^2 w}{dz^2} \quad (32)$$

whereas (31) implies that

$$\frac{1}{\phi_0} \frac{d^2 \phi}{ds^2} = \frac{1}{K^2 \Delta L} \frac{d^2 w}{dz^2} \quad (33)$$

Accordingly we choose

$$\frac{\rho_r}{\alpha^2 A_r \rho_e} = \frac{G_m}{K^2 E_f A_f}$$

or

$$\frac{\rho_r}{\rho_e} = \frac{\alpha^2}{K^2} \frac{G_m}{E_f} \frac{A_r}{A_f} = \frac{E'_m}{E_f} \quad (34)$$

With this resistivity ratio, the analogue relation (31) holds for any arbitrary array of fibers aligned parallel to the z-axis, with any arbitrary distribution of fiber fractures. The strain and therefore the stress distribution in both fiber and matrix can thus be found by constructing a geometrically similar array of conducting rods with scale factor K in the x and y directions and αK in the z-direction, immersing it in an electrolyte of appropriate resistivity, applying a potential to the ends, and measuring the potential distribution with a probe.

Discussion

Up to this point only simple tension has been considered. Pure bending can be simulated by changing the end boundary conditions. For example, bending in the x-z plane can be simulated by applying a linearly varying potential to the rod ends:

$$\phi(x, y, \pm L/2) = \pm \phi_1 x/a \quad (35)$$

where a is the half-width of the composite. Combined tension and bending is simulated by

$$\phi(x, y, \pm L/2) = \pm \phi_0 \pm \phi_1 x/a \quad (36)$$

Shear can be simulated by taking

$$\phi(\pm 0.5b, y, z) = \pm \phi_2 \quad (37)$$

Debonding without friction between fiber and matrix can be simulated applying insulating tape over the appropriate portion of conducting rod. Debonding with friction can be simulated by using tape with the appropriate resistivity. Cracks in the matrix can be simulated by placing insulating sheet in the appropriate places in the electrolyte.

A number of techniques can be employed for finding the stiffness of a damaged composite. Gottesman, Hashin and Brull [14] bound the stiffness by finding the energy and the complementary energy in tension or in shear, and also use the self-consistent theory to obtain an approximate answer. Alternatively a shear lag approach or numerical techniques such as finite element calculations can be employed. All of these are somewhat laborious. Using the electrical analogue approach the stiffness determination is much simpler. It was pointed out in the previous section that elastic modulus and resistivity are inversely related. As a consequence of this, the ratio of the stiffness of a damaged composite to that of the undamaged composite is simply the ratio of the overall resistance of the undamaged to that of the damaged electrical analogue.

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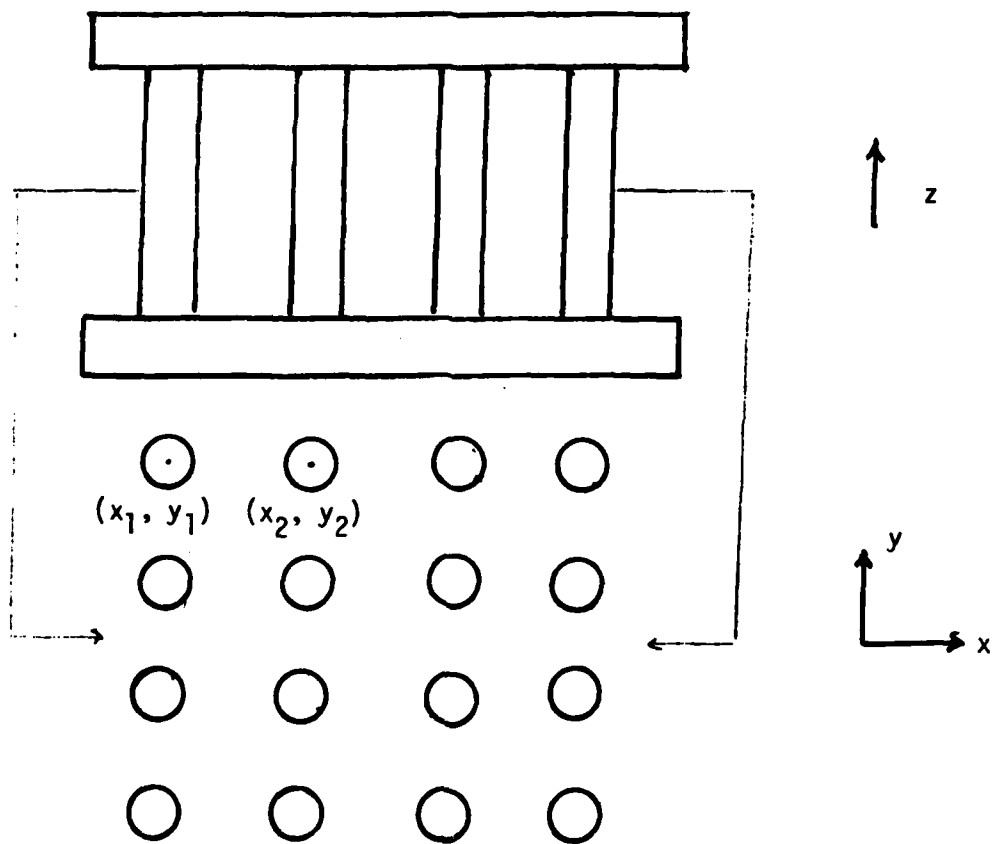


Fig. 1 Coordinate System Employed